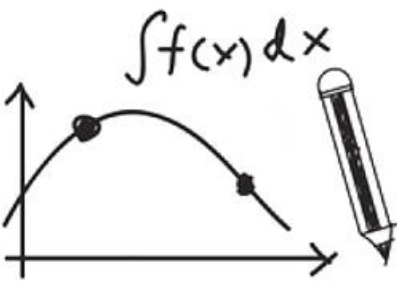




Calculus(I)

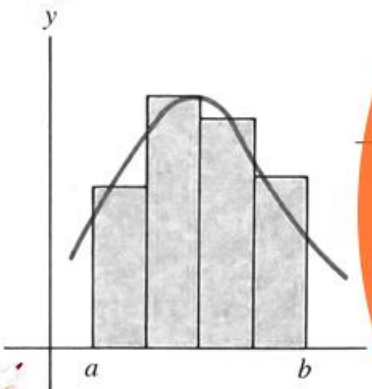
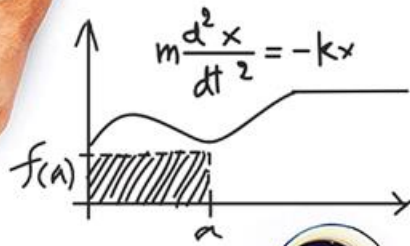
$$x^2 - 3x - 4 = 0$$

$$4x^2 - 3x - 1 = 0$$



$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

$$F = mg = ma = m \frac{d^2h}{dt^2}$$



Gottfried Wilhelm Leibniz

$$\frac{dA}{dt} = \frac{dB}{dt} = -\frac{dC}{dt} = \frac{dD}{dt} = (c_1)T^{\frac{1}{2}}AB - (c_2)T^{\frac{1}{2}}CD$$

$$m \frac{d^2x}{dt^2} = -kx - f \frac{dx}{dt} + A \sin(\omega t)$$

$$y' = \text{and } v' = -ky - fv + A \sin(\omega t)$$

$$m \frac{d^2x}{dt^2} = -kx$$

$$x = A \frac{dT}{dt} - (c_1)(T - T)$$

$$\frac{df(x)}{dx}$$

$$\frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x + \frac{b}{2a} = -\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x + h, f(x + \tau)$$



Rules for Finding Derivatives

Lecturer: Xue Deng

How to find the derivatives of the following expressions?

$$[f(x) + g(x)]', [f(x) - g(x)]', [f(x) \cdot g(x)]', [f(x)/g(x)]'$$

addition

subtraction

multiplication

division



According to the definition of derivative.

Rules for Finding Derivatives

Th A

$$C' = 0, \quad (C \text{ is a constant})$$

Th B

$$x' = 1$$

Th C

$$(x^n)' = nx^{n-1}$$

Th D

$$(kf(x))' = k \cdot f'(x)$$

Th E

$$[f(x) \pm g(x)]' = f'(x) \pm g'(x)$$

Th F

$$[f(x) \cdot g(x)]' = f'(x)g(x) + f(x)g'(x)$$

Th G

$$[f(x)/g(x)]' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)} \quad (g(x) \neq 0)$$

Rules for Finding Derivatives

According to Th G,

$$\left[\frac{f(x)}{g(x)} \right]' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)} \quad (g(x) \neq 0)$$


we have $\left(\frac{\mathbf{1}}{\mathbf{g(x)}} \right)' = \frac{-g'(x)}{g^2(x)}$

Corollary: $(f + g + h)' = f' + g' + h'$

$$(fgh)' = f'gh + fg'h + fgh'$$

Example 1

Find the derivative of $y = x^3 - 2x^2 + \sin x$

 $y' = 3x^2 - 4x + \cos x.$

Example 2

Find the derivative of $y = \sin 2x \cdot \ln x$



$$\because y = 2 \sin x \cdot \cos x \cdot \ln x$$

$$y' = 2 \cos x \cdot \cos x \cdot \ln x + 2 \sin x \cdot (-\sin x) \cdot \ln x + 2 \sin x \cdot \cos x \cdot \frac{1}{x}$$

$$= 2 \cos 2x \ln x + \frac{1}{x} \sin 2x.$$

Example 3

Find the derivative of $y = \frac{x-1}{x+1}$ $\left(\frac{1}{g(x)}\right)' = \frac{-g'(x)}{g^2(x)}$

Method
1

$$y' = \frac{(x-1)'(x+1) - (x-1)(x+1)'}{(x+1)^2} = \frac{2}{(x+1)^2}.$$

Method
2

$$y = \frac{x-1}{x+1} = 1 - \frac{2}{x+1},$$
$$y' = (1)' - \left(\frac{2}{x+1}\right)' = -2 \frac{-1}{(1+x)^2} = \frac{2}{(x+1)^2}.$$

Summary

Th E

$$[f(x) \pm g(x)]' = f'(x) \pm g'(x)$$

Th F

$$[f(x) \cdot g(x)]' = f'(x)g(x) + f(x)g'(x)$$


Th G

$$[f(x)/g(x)]' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)} \quad (g(x) \neq 0)$$

Questions and Answers

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

? Find the derivative of $y = \tan x$

 $y' = (\tan x)' = \left(\frac{\sin x}{\cos x}\right)' = \frac{(\sin x)' \cos x - \sin x (\cos x)'}{\cos^2 x}$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x.$$

Namely,

$$(\tan x)' = \sec^2 x.$$

$$(\cot x)' = -\csc^2 x.$$

Similarly,

$$(\sec x)' = \sec x \cdot \tan x.$$

$$(\csc x)' = -\csc x \cdot \cot x.$$

Questions and Answers



Find the derivative of $(3x^2 - 5)(2x^4 - x)$ by use of the Product Rule
Check the answer by doing the problem in a different way



$$\begin{aligned} \left[(3x^2 - 5)(2x^4 - x) \right]' &= (3x^2 - 5)'(2x^4 - x) + (3x^2 - 5)(2x^4 - x)' \\ &= (6x)(2x^4 - x) + (3x^2 - 5)(8x^3 - 1) \\ &= 24x^5 - 3x^2 - 40x^3 + 5 + 12x^5 - 6x^2 \\ &= 36x^5 - 40x^3 - 9x^2 + 5 \end{aligned}$$


To check, we first multiply and then take the derivative.

$$(3x^2 - 5)(2x^4 - x) = 6x^6 - 10x^4 - 3x^3 + 5x$$

$$\text{Thus, } \left[(3x^2 - 5)(2x^4 - x) \right]' = (6x^6)' - (10x^4)' - (3x^3)' + (5x)' = 36x^5 - 40x^3 - 9x^2 + 5$$

Questions and Answers

? Find $\left(\frac{3x-5}{x^2+7}\right)'$

 $\left[\frac{3x-5}{x^2+7}\right]' = \frac{(x^2+7)(3x-5)' - (3x-5)(x^2+7)'}{(x^2+7)^2}$

$$= \frac{(x^2+7)(3) - (3x-5)(2x)}{(x^2+7)^2}$$
$$= \frac{-3x^2 + 10x + 21}{(x^2+7)^2}.$$

Rules for Finding Derivatives

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